Optimal Portfolio Construction Using Sharpe's Single-index Model: A Case Study of Amman Stock Exchange

Dima Waleed Alrabadi*, Said Sami Al-Hallaq* and Areen Ghaleb Bani-Hamad*

Received on: 02/06/2020 DOI:https://doi.org/10.47017/31.4.10 Accepted on: 08/07/2021

Abstract

The main objective of this study is to construct an optimal portfolio using Sharpe's single-index model (SIM). The study generates a portfolio using daily returns of the Jordanians companies listed in Amman Stock Exchange (ASE) over the period of January 1, 2013 to December 31, 2017. The suggested technique formulates a unique cut-off rate, selects stocks having excess return to beta ratio higher than this cut-off point and defines the percentage of investment in each of the selected stocks. Among the thirty sample companies, only ten were selected for optimal portfolio using SIM. The results show that according to Sharpe's model, the optimal portfolio achieves a daily rate of return of 0.35% and a daily risk measured by the variance of returns of 0.075%.

Keywords: Portfolio construction, Sharpe's single-index model, Optimal portfolio, Cut-off rate, Amman Stock Exchange.

Introduction

Securities available to an investor for investment are numerous and of various types. Investing in securities, such as shares, debentures and bonds, is profitable as well as risky. It is rare to find investors investing their entire wealth in a single security. Instead, they hold a group or portfolio of securities. Portfolio construction can be simply viewed as an issue of selecting securities to include in a portfolio and then determining the appropriate weighting: the proportion of each security to be invested in the portfolio. Portfolio selection was introduced first by "Harry Markowitz" in his pioneering work on portfolio analysis described in his (1952) Journal of Finance article, "Portfolio Selection". His method of portfolio selection has come to be known as the Markowitz model. The main assumption underlying the Markowitz model to portfolio analysis is that investors are essentially risk-averse, meaning that given a choice between two securities with equal rates of return, an investor will select the security with the lower level of risk. This implies that investors must be compensated with a higher return in order to accept higher risk. Markowitz (1952) recognized that by combining assets that are not perfectly correlated, an investor could reduce his or her investment risk without reducing the expected returns. He tried to show that the variance of the rates of return is a worthwhile measure of portfolio risk. His work puts emphasis on the importance of diversification to reduce the risk of a portfolio and show how to diversify such risk effectively.

One of the problems with the Markowitz model is the large number of input data required for calculations. An investor must obtain estimates of return and variance of returns for all securities included in the portfolio, where he/she would need N return estimates, N variance estimates and N(N - 1)/2 covariance estimates, resulting in a total of 2N + [N(N-1)/2] estimates. The second difficulty with Markowitz model is the complexity of computations required. The computations required are numerous and complex in nature. With a given set of securities, an infinite number of portfolios can be constructed. The expected returns and variances of returns for each possible portfolio have to be computed. The identification of efficient portfolios requires the use of quadratic programming which is a complex procedure (Kevin, 2006). Due to these difficulties, security analysts do not like to perform their tasks taking a huge load of data-inputs of this model. They seek a more simplified model for conducting their tasks comfortably. Simplification is needed in the amount and the type of input

© 2022 AYHSS Publishers / Yarmouk University, Irbid, Jordan. All Rights Reserved.

* Faculty of Business, Yarmouk University, Irbid, Jordan

data required to perform portfolio analysis. It is also needed in the computational procedure used to select the optimal portfolio. To this direction, in 1963, William F. Sharpe, most noted for his development of the CAPM, developed the Single-index Model (SIM). SIM is a statistical model for representing a return-generating process. It assumes that all the numbers in the covariance matrix can be accounted for by the fact that all the stocks are responding to the pull of a single factor. SIM is the simplest and most widely used simplification and may be considered as being at one extreme of continuum with the Markowitz model at the other extreme point. The primary objective of this study is to construct an optimal portfolio by using Sharpe's Single-index Model and using data of the Jordanian companies listed in Amman Stock Exchange (ASE) over the period (2013-2017). This study is important for investors who are faced with the problem of deciding which securities to hold and how much to invest in each. The importance of this research appears from the importance of optimal portfolio which all investors wish to attain. Also, the study is important for academicians to get a better understanding of the empirical application of SIM and provide suggestions and recommendations for researchers who are interested in this field.

Amman Stock Exchange ASE

ASE is an emerging stock market that has been established in 1976 as Amman Financial Market (AFM). The Securities Law of 1997 separated the functions of the Amman Financial Market (AFM) and created the Jordan Securities Commission (JSC), the Amman Stock Exchange (ASE) and the Securities Depository Center (SDC). Amman Stock Exchange (ASE) has started operations in 1999 as a non-profit independent institution; authorized to function as a regulated market for trading securities in Jordan. The exchange is governed by a seven-member board of directors. A chief executive officer oversees day-to-day responsibilities and reports to the board. The members of ASE are Jordan's 68 brokerage firms. Amman Stock Exchange became a public shareholding company completely owned by the government under the name "The Amman Stock Exchange Company (ASE Company)" on February 20, 2017.

ASE applies an automated order-driven system. Investors cannot trade unless through brokers. The brokers trade on the system remotely (from their offices) or use trading screens located in the ASE headquarters. They trade on behalf of their customers and for their own inventory. Trading takes place on a continuous basis. Specifically, the brokers enter buy and sell orders into the trading system. The system arranges orders according to price/time priority. In particular, bid prices are ordered in priority from highest to lowest, while ask prices are arranged from lowest to highest. Again, the system matches orders and executes them according the price/time priority. Trade size is not a priority in execution. The system keeps the traders anonymous until the order is executed.

Companies in ASE are classified into three main sectors; financial, industrial and services. According to the formal statistics of 2020 as shown on its website, the number of companies in ASE is 179, the market capitalization is 12908 million JD which accounts for 41.5% of the GDP and the trading volume is 1048.8 million JD.

Literature Review

Many studies have investigated the optimal portfolio construction using different techniques in both developed and emerging stock exchanges around the world. Mroua and Abid (2014) investigated the performance of a revision procedure for domestic and international portfolios and provided an empirical selection strategy for optimal diversification from an American investor's point of view. This paper considers the impact of estimation errors on the optimization processes in financial portfolios. It introduces the concept of portfolio resampling using Monte Carlo method. Statistical inferences methodology is applied to construct the sample acceptance regions and confidence regions for the resampled portfolios needing revision. Tracking error variance minimization (TEVM) problem is used to define the tracking error efficient frontiers (TEEFs) referring to Roll (1992). The data analyzed in this paper consists of daily continuously compounded returns, for stocks and market indices in the period from August 3, 1997, to August 31, 2011. Daily closing prices of 27 American stocks obtained from CRSP are used to form various domestic diversified portfolios. The main

finding is that the global portfolio diversification benefits exist for domestic investors, in both the mean-variance analysis and tracking error analysis. Through TEEF, the dynamic analysis indicates that domestic dynamic diversification outperforms international major and emerging diversification strategies. Portfolio revision appears to be of no systematic benefit.

Sinha *et al.* (2015) developed an algorithm to create an optimum portfolio from a large pool of stocks listed in a single market index S&P 500 Index: USA (for example) using Genetic Algorithm. The algorithm selects stocks on the basis of a priority index function designed on company fundamentals and then genetically assigns optimum weights to the selected stocks by finding a genetically suitable combination of return and risk on the basis of historical data. The effect of genetic evolution on portfolio optimization has been demonstrated by developing a MATLAB code to implement the genetic application of reproduction, crossover and mutation operators. The effectiveness of the obtained portfolio has been successfully tested by running its performance over a six-month holding period. The input shall consist of the stocks from SP 500 Index: US comprising of the daily closing prices, EPS ratios, PEG values, weighted average cost of capital, market capitalization/revenue and return on invested capital in the US Market from the duration December 2011 to December 2012. Based on the inputs from Bloomberg, it is found that genetic algorithm is successful in providing the optimum weights to stocks which were initially screened through a predetermined priority index function. The constructed portfolio beats the market for the considered holding period by a significant margin. The analysis of the portfolio shows an annual average return (AAR) of 26.01% from the historical data (December 2011 to December 2012). When tested for a six-month holding period (Jan. 2013- June 2013), the portfolio performed reasonably well giving a %AAR of 15.98% whereas the market return for the same period was 10.14%. Thus, the above two-stage technique involving genetic algorithm is found to be highly objective, useful and effective for portfolio construction and optimization .

Solimanpur *et al.* (2015) presented a multi-objective model to the optimum portfolio selection using a genetic algorithm and analytic hierarchy process (AHP). The proposed approach solves the problem in two stages. In the first stage, the portfolio selection problem is formulated as a zero-one mathematical programming model to optimize two objectives; namely, return and risk. A genetic algorithm (GA) with multiple fitness functions called as Multiple Fitness Functions Genetic Algorithm is applied to solve the formulated model. The proposed GA results in several non-dominated portfolios are in the efficient frontier. A decision-making approach based on AHP is then used in the second stage to select the portfolio from among the solutions obtained by GA. The GA is applied for data collected from Data Stream, which includes 94 firms of S&P 100 index of the US stock market, during the period (2001-2010). The study finds that the proposed decision-making system enables an investor to find a portfolio which suits his/her expectations at most. The main advantage of the proposed method is to provide preliminary information about the optimal portfolios lying on the efficient frontier and thus help investors decide the appropriate investment alternatives.

Biswas (2015) proposed a multi-objective linear programming portfolio selection model that ensures a no-dominated solution on the efficient frontier based on the outputs of the single-index model. Taking Dow Jones Industrial Average (DJIA) as the market index and considering monthly indices along with the monthly prices of 28 securities over the period March 1999 to March 2015, this model solves a practical portfolio selection problem in a multi-objective framework. The proposed model also shows its superiority over Sharpe's single-index model.

Zaimovic *et al.* (2017) identified the relationships between returns of companies traded in South-East European (SEE) equity markets. A Markowitz mean-variance (MV) portfolio optimization method is used to identify possibilities for diversification among these markets and world-leading capital markets. This research also offers insight into the level of integration of South-East European equity markets. Principal component analysis (PCA) is used to determine components that describe the strong patterns and co-movements of the dataset. The sample of the study consists of 47 stocks and 23 indices observed over the period from 1st January 2006 to 1st April 2016. The findings show that PCA analysis substantially simplifies the asset selection process in portfolio management. There is a rather

limited diversification benefit from spreading out the investments from the SEE markets to the world leading capital markets, or *vice versa*, due to the high integration of the SEE market with leading world markets. National capital markets are quite inefficient from the mean-variance standing point of view, while the Macedonian capital market was the best performing in the observed period.

Mrcela *et al.* (2017) proposed a new algorithm for portfolio optimization, based on statistical arbitrage, that uses a multi-criteria decision-making approach to obtain the most preferred assets. A preference flow graph of financial assets is constructed at each time step, with the aid of statistical arbitrage algorithm that describes preferences among the assets. Then, the individual preferences for each asset are obtained by using the potential method and the most preferred assets are selected into the portfolio in accordance to them. A consistency measure of the preference flow graph is also obtained using the same method and it measures the reliability of the decision-making. The proposed method has been tested on a selection of S&P 500 constituent stocks from 1980 to 2004. The results indicate that the proposed method performs well in the considered market, which is indicated by high Sharpe ratios of the constructed portfolios. The algorithm performs better when provided with a larger number of assets, showing that the increased number of considered assets provides more insight into the market behavior.

Studies in emerging markets also used the SIM. Mandal (2013) investigated the idea hidden in Single-index Model (SIM) and constructed an optimal portfolio using this model, basing on daily stock prices of ten selected public-sector enterprises along with daily indices of BSE Sensex as market performance index for the period from April 2001 to March 2011. After formulating the cut-off rate, securities with "excess-return to beta" ratio values greater than or equal to the cut-off point were selected. Then, to arrive at the optimal portfolio, the proportion of investment in each of the selected securities in the optimal portfolio was computed on the basis of beta value, unsystematic risk, excess return to beta ratio and the cut-off rate of the security concerned. The study found that Sharpe's Single-index Model gives an easy mechanism to construct an optimal portfolio of stocks and simplifies the portfolio problems found in Markowitz's model. The results also indicated that there is a significant difference between the total risk of the optimal portfolio under SIM and that under Markowitz's model.

Sarker (2013) studied the applicability of using Single-index Model in constructing an optimal portfolio, considering a sample of monthly prices of 164 companies listed in Dhaka Stock Exchange (DSE) and DSE all-share price index over the period of July 2007 to June 2012. The study found that portfolio beta is considerably lower than the market beta and portfolio return is much higher than the portfolio variance. The framework of Sharpe's single-index model for optimal portfolio construction is found to be very simple and useful. Portfolio return and portfolio risk have been found out to be, respectively, 6.17% and 8.76%.

Das and Agarwal (2014) concentrated on the construction of an optimal portfolio of banking stocks using Sharpe's index model. The data has been collected from the website of National Stock Exchange and Reserve Bank of India over the period (2009 - 2014). The sample size under this study is limited to twelve stocks. The twelve companies are ranked on the basis of excess return to beta ratio. The cut-off point is calculated for each stock and the highest value of cut-off point was taken into calculations in determining the percentage of money to be invested in each banking stock. The outcomes of this study guide investors in their decision of selecting best banking stocks from the Bank Nifty Index.

Ramanathan and Jahnavi (2014) constructed an optimal equity portfolio using Sharpe's index model taking media and entertainment sector into consideration for constructing the optimum portfolio. The data of the study consists of twenty Indian companies over a period of 5 years from April 2007 to March 2013. Excess return to beta ratio has been calculated and the companies are ranked based on that ratio. The cut-off point is calculated based on the highest value and cut-off point is used to calculate the proportion of money to be invested in each stock. The study concluded that the industry is performing well over the period of the study. In the securities market, stock with high risk will yield high return. Thus, PVR Company has high risk and yields higher return compared to the

other stocks in the portfolio, where Dish TV company yields lower return among other companies. The study also found that media and entertainment industry is the growing and emerging sector in the securities market under investigation.

Chou *et al.* (2017) used GA combined with the Sharpe ratio to find the best portfolio which has low risk and high return and utilized a novel method, funds standardization, to calculate portfolio risk. Moreover, this paper used sliding windows to avoid the over-fitting problem. The source of stock prices in the study is the Taiwan Economic Journal (TEJ) during the period from 2010 to June 2016. The constituent stocks of Taiwan 50 ETF are chosen as investment target. The experimental results showed that the portfolio can spread the risk effectively and that the portfolio risk can be assessed accurately by utilizing the funds standardization. Comparing with the traditional method, this method can identify the optimal portfolio efficiently and establish a portfolio that has lower risk and stable return.

In the Jordanian context, Alrabadi (2016) utilized the mean-variance optimization framework of Markowitz (1952) and the generalized reduced gradient (GRG) nonlinear algorithm to find the optimal portfolio that maximizes return while keeping risk at minimum. This study applied the portfolio optimization concept of Markowitz (1952) and the GRG nonlinear algorithm to a portfolio consisting of the 30 leading stocks from the three different sectors (industrial, services and financial) in Amman Stock Exchange over the period from 2009 to 2013. The study found that the selected portfolio achieves a monthly return of 5 percent whilst keeping risk at minimum. However, if the short-selling constraint is relaxed, the monthly return will be 9 percent. Moreover, the GRG nonlinear algorithm enables to construct a portfolio with a Sharpe ratio of 7.4.

Al Saadi (2017) investigated the ability of a hybrid genetic algorithm (HGA) and analytic hierarchy process (AHP) in selecting the optimal portfolio. The study used daily returns of the companies listed in Amman Stock Exchange over the period from January 1, 2015, to December 31, 2015. The study used the following variables in order to achieve the purpose of the study: return, risk, beta, liquidity ratio, Sharpe ratio, Treynor's ratio and Jensen's alpha. The attempted mathematical model includes two objective functions; namely, return and risk of a portfolio. The study found that HGA can identify portfolios that are on the efficient frontier. HGA has no restrictions on the number of assets. It also found that AHP can select the optimum portfolio among the portfolios obtained by HGA.

Overall, previous studies have developed many techniques of portfolio optimization in different stock markets around the world. The optimal portfolio construction using Sharpe's single-index model is found to be simple and effective in developed and emerging stock exchanges. Thus, simplicity and efficiency at the same time are what distinguishes it from other more sophisticated models. To the best of the authors' knowledge, this is the first study in Jordan that uses this model in order to find the optimum portfolio in ASE using daily data over the period (2013-2017). The results of this study are expected to be vital to both academicians and investors in ASE.

Data and Methodology

The study population consists of all securities listed in Amman Stock Exchange (ASE) during the period of January 1, 2013 to December 31, 2017. The sample consists of the largest 30 companies in ASE according to their market capitalization. The choice of the largest companies is based on including the blue-chip stocks in the constructed portfolio and excluding the small companies. Stocks are selected from the three different sectors (industrial, services and financial), 10 stocks each. The data is collected on a daily basis over the study period.

To attain the objective of the study, we use two main variables; the return of individual stocks (R_{it}) and market return (R_{mt}).

Stock's return

$$R_{it} = Ln\left(\frac{p_{it}}{p_{i,t-1}}\right) \quad (1)$$

where

R_{it} is the return of stock i on day t.

p_{it} is the closing price of stock i on day t.

$P_{i,t-1}$ the closing price of stock i on day t-1.

Market return

$$R_{mt} = Ln\left(\frac{Index_t}{Index_{t-1}}\right) \quad (2)$$

where

R_{mt} is the market return on day t.

$Index_t$ is the general free float index value on day t.

$Index_{t-1}$ is the general free float index value on day t-1.

The Construction of the Optimal Portfolio

The first step to construct an optimum portfolio using Sharpe's Single-index Model is to select securities on the basis of the following criteria:

- The return on the investment is greater than the risk-free return.
- The beta value for that security is positive.

For each security selected in the portfolio, return is then calculated following Equation (1).

After selecting these securities in the portfolio, the next step is to construct an optimal portfolio. The construction of an optimal portfolio is a simple function if a single number gauges the desirability of including a security in the optimal portfolio. For Sharpe's Single-index Model, such a number exists. In this case, the desirability of any security is directly related to its "excess return-to-beta" ratio given by the following equation:

$$\text{Excess return-to-beta ratio} = \frac{R_i - R_f}{\beta_i} \quad (3)$$

where

R_i is the return on stock i

R_f is the risk-free rate of return

β_i is a constant that measures the expected change in R_i given a certain change in R_m.

So, "excess return-to-beta" ratio is calculated for each security in the portfolio. This ratio measures the additional return on a security (excess of the riskless assets return) per unit of systematic risk or non-diversifiable risk. Securities are ranked in descending order (from highest to lowest), according to their "excess return-to-beta" ratio. Further, the number of stocks selected in the optimum portfolio depends on a unique cut-off rate C* such that all stocks with "excess return-to-beta" ratios greater than or equal to this unique cut-off C*are included and all stocks with lower ratios are excluded.

The value of C* is computed from the attributes of all securities that belong to the optimum portfolio. For a portfolio of i stocks, C_i is given by:

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \frac{(R_i - R_f)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{n} \left(\frac{\beta_i^2}{\sigma_{ei}^2}\right)} \quad (4)$$

After determining the qualified securities to be selected, the investor should find out how much should be invested in each security, The percentage invested in i^{th} security is denoted by X_i and is calculated using the expression:

$$X_1 = \frac{z_i}{\sum_{i=1}^{N} z_j} \quad (5)$$

where,

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{R_i - R_f}{\beta_i} - C^*\right) \quad (6)$$

The first expression indicates the weights on each security and they sum up to one. The second expression determines the relative investment in each security.

Finally, expected return of the optimum portfolio *Rp* is calculated using the following equation (Strong, 2008):

$$Rp = \sum_{i=1}^{N} XiRi \quad (7)$$

Portfolio variance $\sigma^2 p$ is calculated using the following equation (Strong, 2008):

$$\sigma^2 p = (\sum_{i=1}^{N} Xi\beta i)^2 \sigma^2 m + \sigma^2 ep \quad (8)$$

Results of Analysis

Table (1) shows the descriptive statistics of the returns of sample companies (mean, median, maximum, minimum and standard deviation). It shows that the market return has a daily mean of 0.0001, a median of 0.0000, a maximum of 0.0209, a minimum of -0.0198 and a standard deviation of 0.0043 over the period of (2013-2017).

Table (1): Descriptive statistics of the returns of sample companies.

	Mean	Median	Maximum	Minimum	Std. Dev.
Jordan Islamic Bank	-0.0001	0.0000	0.0796	-0.1945	0.0173
Jordan Kuwait Bank	-0.0012	0.0000	0.0715	-0.1961	0.0191
The Housing Bank for Trade and Finance	0.0004	0.0000	0.0716	-0.2351	0.0147
Bank of Jordan	-0.0009	0.0000	0.2121	-1.2917	0.0541
Arab Bank	0.0006	0.0000	0.8685	-0.1495	0.0368
Al-Bilad Medical Services	-0.0102	0.0000	0.0813	-1.5488	0.1127
Jordan Press Foundation/Al-RA'I	-0.0034	0.0000	1.6386	-0.3399	0.0846
Al-Tajamouat for Touristic Projects Co., PLC	-0.0014	0.0000	0.1054	-0.1719	0.0259
Jordanian Duty-Free Shops	0.0228	0.0000	3.2171	-0.0777	0.2389
The Arab International for Education & Investment	-0.0060	0.0000	0.0720	-2.4400	0.1260
Union Investment Corporation	-0.0011	0.0000	0.2458	-0.6749	0.0412

	Mean	Median	Maximum	Minimum	Std. Dev.
Union Land Development Corporation	0.0007	0.0000	0.7017	-0.1380	0.0434
Arab East Investment	-0.0037	0.0000	0.2076	-0.8837	0.0507
Al-Dawliyah for Hotels & Malls	-0.0013	0.0000	0.0335	-0.1118	0.0145
Jordan Telecom	-0.0010	0.0000	1.8822	-0.3189	0.0779
Alia- The Royal Jordanian Airlines PLC.	-0.0065	0.0000	0.0690	-1.1763	0.0555
First Jordan Investment Company PLC	-0.0027	0.0000	0.1625	-1.4816	0.0677
Jordan Masaken for Land & Industrial Development Projects	0.0142	0.0000	1.9347	-0.3830	0.1372
Afaq for Energy Co., PLC	-0.0014	0.0000	0.0932	-1.0315	0.0445
Dar Al Dawa Development & Investment	-0.0020	0.0000	0.0834	-0.3049	0.0307
Jordan Phosphate Mines	-0.0033	-0.0031	1.9390	-0.2726	0.0771
The Jordan Cement Factories	-0.0023	0.0000	0.3086	-0.6140	0.0438
The Arab Potash	0.0086	0.0000	3.6490	-0.1245	0.1782
Al-Eqbal Investment Company, LTD	0.0037	0.0000	0.2906	-0.5533	0.0449
Universal Modern Industries	0.0040	0.0000	0.2850	-0.1771	0.0332
Union Tobacco & Cigarette Industries	-0.0023	0.0000	0.8868	-0.4515	0.0659
The Arab Pesticides & Veterinary Drugs MFG Co.	-0.0005	0.0000	0.0694	-0.6396	0.0335
Hayat Pharmaceutical Industries Co.	0.0003	0.0000	0.1232	-0.1671	0.0262
United Cable Industries	-0.0022	0.0000	0.0813	-0.2274	0.0296
Jordan Petroleum Refinery	0.0011	0.0000	2.7166	-0.3502	0.1044
Market	0.0001	0.00	0.0209	-0.0198	0.0043

Table 2 shows the variables needed to construct the optimal portfolio using SIM. Several statistics such as average daily return, variance, standard deviation of daily return, standard deviation of market return, beta, systematic risk and unsystematic risk of all the thirty sampled securities have been calculated on the basis of the collected data. Table 2 shows that Al-Bilad Medical Services has the lowest daily mean return of -0.0102 and Jordanian Duty-Free Shops has the highest mean return of 0.0228, the variances ranging from 0.0002 for the Housing Bank for Trade and Finance to 0.0571 for Jordanian Duty-Free Shops, standard deviation ranging from 0.0145 for Al-Dawliyah for Hotels & Malls to 0.2389 for Jordanian Duty-Free Shops. This table also shows the beta values of the sampled securities. Al-Eqbal Investment Company LTD has the highest beta value of 1.8425, which means that it is highly volatile. Universal Modern Industries has a negative beta of -0.1204, which represents lower volatility.

Table (2): Variables needed to construct optimal portfolio using Sharpe's single-index model.

	Avg. daily return	Standard deviation	Variance	Beta	Systematic risk ($\beta^2\sigma^2rm$)	Unsystematic risk (σ^2ei)
Jordan Islamic Bank	-0.0001	0.0173	0.0003	0.7562	1.07E-05	0.0003
Jordan Kuwait Bank	-0.0012	0.0191	0.0004	0.7206	9.7E-06	0.0004
The Housing Bank for Trade and Finance	0.0004	0.0147	0.0002	0.3429	2.2E-06	0.0002
Bank of Jordan	-0.0009	0.0541	0.0029	0.6450	7.77E-06	0.0029
Return Arab Bank	0.0006	0.0368	0.0014	1.0148	1.92E-05	0.0013
Al-Bilad Medical Services	-0.0102	0.1127	0.0127	0.7419	1.03E-05	0.0127
Jordan Press Foundation/Al-RA'I	-0.0034	0.0846	0.0072	0.2550	1.21E-06	0.0072
Al-Tajamouat for Touristic Projects Co., PLC	-0.0014	0.0259	0.0007	0.7592	1.08E-05	0.0007
Jordanian Duty-Free Shops	0.0228	0.2389	0.0571	0.7040	9.26E-06	0.0571

	Avg. daily return	Standard deviation	Variance	Beta	Systematic risk ($\beta^2\sigma^2rm$)	Unsystematic risk (σ^2ei)
The Arab International for Education & Investment	-0.0060	0.1260	0.0159	0.6606	8.15E-06	0.0159
Union Investment Corporation	-0.0011	0.0412	0.0017	1.7815	5.93E-05	0.0016
Union Land Development Corporation	0.0007	0.0434	0.0019	1.6645	5.18E-05	0.0018
Arab East Investment	-0.0037	0.0507	0.0026	1.1824	2.61E-05	0.0025
Al-Dawliyah for Hotels & Malls	-0.0013	0.0145	0.0026	0.5270	5.19E-06	0.0026
Jordan Telecom	-0.0010	0.0779	0.0061	1.1436	2.44E-05	0.0060
First Jordan Investment	-0.0065	0.0555	0.0031	1.3806	3.56E-05	0.0030
Company PLC	-0.0027	0.0677	0.0046	0.7765	1.13E-05	0.0046
Jordan Masaken for Land & Industrial Development Projects	0.0142	0.1372	0.0188	0.0655	8.01E-08	0.0188
Afaq for Energy Co., PLC	-0.0014	0.0445	0.0020	1.5249	4.34E-05	0.0019
Dar Al Dawa Development & Investment	-0.0020	0.0307	0.0009	0.8279	1.28E-05	0.0009
Jordan Phosphate Mines	-0.0033	0.0771	0.0060	1.7590	5.78E-05	0.0059
The Jordan Cement Factories	-0.0023	0.0438	0.0019	0.5448	5.54E-06	0.0019
The Arab Potash	0.0086	0.1782	0.0317	1.5438	4.45E-05	0.0317
Al-Eqbal Investment Company, LTD	0.0037	0.0449	0.0020	1.8425	6.34E-05	0.0020
Universal Modern Industries	0.0040	0.0332	0.0011	0.1204	2.71E-07	0.0011
Union Tobacco & Cigarette Industries	-0.0023	0.0659	0.0043	0.7948	1.18E-05	0.0043
The Arab Pesticides & Veterinary Drugs MFG Co.	-0.0005	0.0335	0.0011	0.5634	5.93E-06	0.0011
Hayat Pharmaceutical Industries Co.	0.0003	0.0262	0.0007	0.5424	5.5E-06	0.0007
United Cable Industries	-0.0022	0.0296	0.0009	1.2487	2.91E-05	0.0008
Jordan Petroleum Refinery	0.0011	0.1044	0.0109	1.3375	3.34E-05	0.0109
Market	0.0001	0.0043	0.0000			

Table 3 shows the calculations of the cut-off rate. It can be noticed from Table 3 that the unique cut-off has been derived as -0.00021. Thus, only 10 securities having excess return to beta ratio above -0.00021 are qualified to the optimum portfolio. These securities are: Jordan Islamic Bank, the Housing Bank for Trade and Finance, Arab Bank, Jordanian Duty-Free Shops, Union Land Development Corporation, Jordan Masaken for Land & Industrial Development Projects, the Arab Potash, Al-Eqbal Investment Company , Hayat Pharmaceutical Industries and Jordan Petroleum Refinery.

Table (3): Calculations of cut-off rate.

	Excess return to beta ratio	R*B/ σ^2ei	B^2/σ^2ei	C*
Jordan Islamic Bank	-9.8E-05	-0.19458	1985.144	**-0.00021**
Jordan Kuwait Bank	-0.0017	-2.48109	1458.875	
The Housing Bank for Trade and Finance	0.00122	0.666923	546.447	
Bank of Jordan	-0.00142	-0.20184	142.5972	
Return Arab Bank	0.000573	0.443372	773.2154	
Al-Bilad Medical Services	-0.0137	-0.59406	43.37373	

	Excess return to beta ratio	R*B/ σ^2ei	B^2/σ^2ei	C*
Jordan Press Foundation/Al-RA'I	-0.01337	-0.12145	9.082807	
Al-Tajamouat for Touristic Projects Co., PLC	-0.00181	-1.5745	870.2277	
Jordanian Duty-Free Shops	0.032411	0.281435	8.683178	
The Arab International for Education & Investment	-0.00905	-0.24879	27.49879	
Union Investment Corporation	-0.00063	-1.22465	1940.631	
Union Land Development Corporation	0.000426	0.645127	1514.175	
Arab East Investment	-0.00314	-1.72673	550.4951	
Al-Dawliyah for Hotels & Malls	-0.00255	-0.27623	108.4706	
Jordan Telecom	-0.0009	-0.19542	216.3017	
Alia- the Royal Jordanian Airlines PLC.	-0.00468	-2.92214	624.9621	
First Jordan Investment Company PLC	-0.00342	-0.45217	132.0362	
Jordan Masaken for Land & Industrial Development Projects	0.217314	0.049495	0.227758	
Afaq for Energy Co., PLC	-0.00094	-1.12583	1201.208	
Dar Al Dawa Development & Investment	-0.00241	-1.77477	735.549	
Jordan Phosphate Mines	-0.00186	-0.97472	525.0524	
The Jordan Cement Factories	-0.00429	-0.66411	154.8415	
The Arab Potash	0.005595	0.420689	75.19161	
Al-Eqbal Investment Company, LTD	0.001991	3.46546	1740.434	
Universal Modern Industries	-0.03332	-0.4388	13.16897	
Union Tobacco & Cigarette Industries	-0.00295	-0.43089	146.0206	
The Arab Pesticides & Veterinary Drugs MFG Co.	-0.00096	-0.27324	284.0533	
Hayat Pharmaceutical Industries Co.	0.000637	0.275857	433.3024	
United Cable Industries	-0.00175	-3.21688	1840.689	
Jordan Petroleum Refinery	0.000797	0.13111	164.542	

Table 4 shows the proportion of investment to be made in each security. From Table 4, it can be observed that the highest proportion is for Al-Eqbal Investment Company, LTD with 49.83% and the smallest proportion is for Jordan Masaken for Land & Industrial Development Projects with 0.65%.

Table (4): Calculations of the weights of securities selected in the optimal portfolio.

	z value	Weight
Jordan Islamic Bank	0.2127897	0.02774
The Housing Bank for Trade and Finance	0.7790582	0.1015606
Arab Bank	0.6020409	0.078484
Jordanian Duty-Free Shops	0.2832165	0.036921
Union Land Development Corporation	0.9558461	0.1246073
Jordan Masaken for Land & Industrial Development Projects	0.0495418	0.0064584
The Arab Potash	0.436119	0.0568539
Al-Eqbal Investment Company, LTD	3.8226095	0.498328
Hayat Pharmaceutical Industries Co.	0.3647734	0.0475531
Jordan Petroleum Refinery	0.1648749	0.0214936

Table 5 shows the calculations for return and variance of the optimal portfolio. It is seen that the portfolio daily return is 0.35%. This means that the investor will get 0.35% return from investing in a portfolio consisting of 10 securities having excess return to beta ratio higher than the cut-off rate. In against, he/she will afford 0.075% portfolio risk.

Table (5): Return and risk of the optimal portfolio

	Xi * Ri	Xi * beta i	Xi² *unsystematic risk
Jordan Islamic Bank	-2.05608E-06	0.02097696	2.21663E-07
The Housing Bank for Trade and Finance	4.25062E-05	0.03482767	2.21973E-06
Arab Bank	4.56691E-05	0.07964443	8.20371E-06
Jordanian Duty-Free Shops	0.000842475	0.02599312	7.78105E-05
Union Land Development Corporation	8.83662E-05	0.20740404	2.84092E-05
Jordan Masaken for Land &Industrial Development Projects	9.18793E-05	0.00042279	7.84847E-07
The Arab Potash	0.000491057	0.08776874	0.00010245
Al-Eqbal Investment Company, LTD	0.001828241	0.91818485	0.000484398
Hayat Pharmaceutical Industries Co.	1.64219E-05	0.02579473	1.53557E-06
Jordan Petroleum Refinery	2.29062E-05	0.02874717	5.02242E-06
Total	**0.003467466**	**1.42976451**	**0.000711055**
Portfolio return Rp=Sum(Xi * Ri)	**0.35%**		
Portfolio variance $\sigma^2p = Bp^2*\sigma m^2 + \sigma ep^2$ $\sigma p^2 = (1.42976451)^2 * 0.0000186797+0.000711055$	**0.075%**		

Table 6 shows the correlation between securities returns in the optimal portfolio. It shows that each two stocks can be positively correlated or negatively correlated. A positive correlation indicates that the two securities returns move in the same direction; if one security return increases, the other increases as well. Negative correlation indicates that the two securities returns move in opposite directions; if one security return increases, the other decreases. A correlation coefficient of 1 means that for every positive increase in one security return, there is a positive increase of the same proportion in the other. According to modern portfolio theory, diversified portfolio will be benefited if the correlation coefficient between the selected stocks for portfolio is negative.

Table (6): The correlation between securities in the optimal portfolio.

	Jordan Islamic Bank	The Housing Bank for Trade and Finance	Arab Bank	Jordanian Duty-Free Shops	Union Land Development Corporation
Jordan Islamic Bank	1.0000	0.4752	-0.2055	-0.0573	0.4416
the Housing Bank for Trade and Finance	0.4752	1.0000	-0.4141	-0.0973	0.5548
Arab Bank	-0.2055	-0.4141	1.0000	-0.5467	-0.6380
Jordanian Duty-Free Shops	-0.0573	-0.0973	-0.5467	1.0000	0.0142
Union Land Development Corporation	0.4416	0.5548	-0.6380	0.0142	1.0000
Jordan Masaken for Land & Industrial Development Projects	-0.5420	-0.8930	0.5985	0.0639	-0.6123
The Arab Potash	0.5519	0.6885	-0.4667	-0.1910	0.7870
Al-Eqbal Investment Company, LTD	0.5568	0.7062	-0.5930	0.0882	0.6934
Hayat Pharmaceutical Industries Co.	0.1885	0.1820	-0.3347	0.0187	0.4608
Jordan Petroleum Refinery	-0.1031	-0.3305	0.4592	-0.1242	-0.3938

Table (6) continuous

	Jordan Masaken for Land & Industrial Development Projects	The Arab Potash	Al-Eqbal Investment Company, LTD	Hayat Pharmaceutical Industries Co.	Jordan Petroleum Refinery
Jordan Islamic Bank	-0.5420	0.5519	0.5568	0.1885	-0.1031
The Housing Bank for Trade and Finance	-0.8930	0.6885	0.7062	0.1820	-0.3305
Arab Bank	0.5985	-0.4667	-0.5930	-0.3347	0.4592
Jordanian Duty-Free Shops	0.0639	-0.1910	0.0882	0.0187	-0.1242
Union Land Development Corporation	-0.6123	0.7870	0.6934	0.4608	-0.3938
Jordan Masaken for Land & Industrial Development Projects	1.0000	-0.7011	-0.7840	-0.2141	0.4861
The Arab Potash	-0.7011	1.0000	0.7752	0.2508	-0.3367
Al-Eqbal Investment Company, LTD	-0.7840	0.7752	1.0000	0.2804	-0.6380
Hayat Pharmaceutical Industries Co.	-0.2141	0.2508	0.2804	1.0000	-0.2773
Jordan Petroleum Refinery	0.4861	-0.3367	-0.6380	-0.2773	1.0000

Discussion

This study aims to use Sharpe's Single-index Model to construct an optimal portfolio by using daily data from Amman Stock Exchange for the period from 2013 to 2017. The portfolio consists of 10 stocks that have excess return to beta ratio higher than the cut-off point. We find that according to Sharpe's model, the optimum portfolio daily return is 0.35%, while its risk is 0.075%.

Our results are similar to those of previous studies that constructed an optimal portfolio by using Sharpe's Single-index Model, such as (Mandal, 2013; Sarker, 2013; Ramanthan and Jahnavi, 2014; Singh and Gautam, 2014; Mary *et al.,* 2015; Poormina and Remesh, 2015; Shah, 2015). All securities which have excess return to beta ratio greater than the cut-off point are included in the portfolio. Such portfolio is the optimum portfolio and the securities included in the portfolio are the most efficient securities. Sarker (2013) constructed an optimal portfolio consisting of 33 securities (out of a sample of 164 securities) listed in Dhaka Stock Exchange (DSE), over the period from July 2007 to June 2012. The study found that the framework of Sharpe's single-index model for optimal portfolio construction is very simple and useful. Portfolio monthly return and portfolio risk measured by the standard deviation of returns have been found out to be respectively 6.17% and 8.76%. Shah (2015) built an ideal portfolio using Sharpe Index Model and CAPM model for BSE Top 15 securities and compared the portfolios prepared through Sharpe Index Model &CAPM. The researcher collected monthly data on BSE top 15 securities on basis of market capitalization from January 2000 to March 2015. According to Sharpe model, portfolio monthly return is 1.89%, while portfolio risk is 8.86% measured by the standard deviation of returns and 78.43% measured by the variance of returns. Biswas (2015) evaluated the performance of some selected diversified and non-diversified portfolios with respect to risk and return. Each of these portfolios consists of 6 securities that have been randomly selected. According to his study, diversified portfolio return was 8.98%, which is higher than that of the non-diversified portfolio. The return of non-diversified portfolio was 8.38%. Also, risk of the diversified portfolio was 1.47, which is lower than that of the non-diversified portfolio which was 1.55 measured by the standard deviation of returns. The study of Mroua and Abid (2013) aimed to investigate the performance of a revision procedure for domestic and international portfolios and provide an empirical selection strategy for optimal diversification from an American investor's point

of view. This study used Monte Carlo method to introduce the concept of portfolio resampling and apply statistical inferences methodology to construct the sample acceptance regions and confidence regions for the resampled portfolios needing revision. The findings present that through TEEF, the dynamic analysis indicates that domestic dynamic diversification outperforms international major and emerging diversification strategies. Portfolio revision appears to be of no systematic benefit. Solimanpur *et al.* (2015) in their study aimed to construct optimal portfolios using a hybrid genetic algorithm (HGA) and analytic hierarchy process (AHP). The attempted mathematical model includes two objective functions; namely, risk and return. The proposed GA results in several non-dominated portfolios being in the efficient frontier. A decision-making approach based on AHP is then used to select the portfolio from among the solutions obtained by GA. 12 non-dominated portfolios are obtained. The annual return of optimal portfolios varies between 6.75 and 9.15 percent, while their risk measured by the standard deviation of returns changes between 7.79 and 9.35 percent. Al Saadi (2017) investigated the ability of a hybrid genetic algorithm (HGA) and analytic hierarchy process (AHP) in selecting the optimal portfolio. The data was adopted from Amman Stock Exchange for the year 2015. Using HGA to solve the Markowitz portfolio optimization problem, 10 portfolios are obtained. The annual portfolio return of optimal portfolios varies between 10.552% and 13.429%, while their risk changes between 0.646% and 0.818% measured by the standard deviation of the returns. Alrabadi (2015) found an optimal portfolio that maximizes return while keeping risk to a minimum. This study applied the portfolio optimization concept of Markowitz (1952) and the GRG nonlinear algorithm. It constructed six portfolios with different weights consisting of the 30 leading stocks from the three different sectors in Amman Stock Exchange over the period from 2009 to 2013. The selected portfolios achieve a monthly return of 5 percent while keeping risk measured by standard deviation of the returns at minimum. However, if the short-selling constraint is relaxed, the monthly return will be 9 percent.

Conclusion

This study aims to construct an optimal portfolio by using Sharpe's SIM. The study applies Sharpe's model using daily returns of the companies listed in Amman Stock Exchange over the period of January 1, 2013 to December 31, 2017. Findings can be summarized as follows:

1. Constructing an optimal portfolio using SIM is useful, convenient for investors and simpler than that constructed using Markowitz's Mean-variance Model.
2. According to Sharpe model, optimal portfolio daily return is 0.35%, which means that the investor will get 0.35% daily returns by holding a portfolio containing 10 securities from the three sectors of ASE. In against, the investor will bear a portfolio daily risk of 0.075% measured by the variance of return.
3. The optimum portfolio in ASE over the study period is consisting of (2.77%) of investment made in Jordan Islamic Bank, (10.16%) of investment made in the Housing Bank for Trade and Finance, (7.85%) of investment made in Arab Bank, (3.69%) of investment made in Jordanian Duty-Free Shops, (12.46%) of investment made in Union Land Development Corporation, (0.65%) of investment made in Jordan Masaken for Land & Industrial Development Projects, (5.69%) of investment made in the Arab Potash, (49.83%) of investment made in Al-Eqbal Investment Company , (4.76%) of investment made in Hayat Pharmaceutical Industries and (2.15%) of investment made in Jordan Petroleum Refinery
4. The highest proportion in the optimal portfolio is for Al-Eqbal Investment Company with 49.83% and the smallest proportion is for Jordan Masaken for Land & Industrial Development Projects with 0.65%.
5. Jordanian Duty-Free shops company has the highest mean return of 2.28% and the highest variance of 5.71%, while Al-Bilad Medical Services company has the lowest mean return of -1.02% and the Housing Bank for Trade and Finance has the lowest variance of 0.02%.

بناء المحفظة المثلى باستخدام نموذج شارب للمؤشر الواحد: دراسة حالة لبورصة عمان للأوراق المالية

ديما وليد الربضي، سعيد سامي الحلاق و عرين غالب بني حمد

كلية الاعمال، جامعة اليرموك، إربد، الأردن.

ملخص

تهدف هذه الدراسة إلى اختبار قدرة نموذج شارب للمؤشر الواحد في بناء المحفظة المثلى. تستخدم هذه الدراسة العوائد اليومية للشركات المدرجة في بورصة عمان للأوراق المالية للفترة من 1/كانون الثاني 2013 حتى 31/ كانون الأول 2017. إن النموذج المقترح مبني على حساب نقطة قطع ومن ثم اختيار الأسهم التي تزيد قيمة نسبة فائض العائد إلى بيتا لها عن هذه النقطة، وحساب أوزان هذه الأسهم في المحفظة المثلى. ثم اختيار عشرة أسهم في المحفظة المثلى من أصل ثلاثين سهماً تمثل عينة الدراسة. تبين النتائج باستخدام نموذج شارب للمؤشر الواحد أن المحفظة المثلى تحقق عائداً يومياً يساوي 0.35% وخطورة يومية تساوي 0.075% مقاسه بتباين العوائد.

الكلمات المفتاحية: بناء المحفظة، نموذج شارب للمؤشر الواحد، المحفظة المثلى، نقطة القطع، بورصة عمان للأوراق المالية.

References

Alrabadi, D. (2016). Portfolio optimization using the generalized reduced gradient nonlinear algorithm: An application to Amman Stock Exchange. *International Journal of Islamic and Middle Eastern Finance and Management*, 9(4):570-582.

Al Saadi, M. (2017). *Optimum portfolio selection using a hybrid genetic algorithm and analytic hierarchy process*. Unpublished Master Thesis, Yarmouk University, Jordan.

Biswas, D. (2015). The Effect of Portfolio Diversification Theory: Study on Modern Portfolio Theory of Stock Investment in the National Stock Exchange. *Journal of Commerce & Management Thought*, 6(3): 445-455.

Chou, Y., Kuo, S. and Lo, Y. (2017). Portfolio Optimization Based on Funds Standardization and Genetic Algorithm. *IEEE Access*, 5: 21885 – 21900.

Das, S. and Agarwal, A. (2014). Construction of Optimal Portfolio of Banking Stocks:A Diversification Strategy. *International Journal of Financial Management, 3*(2): 1-6.

Kevin, S. (2006). *Portfolio Management*. 2nd Edition, Prentice-Hall of India.

Mandal, N. (2013). Sharpe's Single-index Model and its applications to construct optimal portfolio: An empirical study. *Great Lakes Herald*, 7(1): 1-22.

Markowitz, H. (1952). Portfolio Selection. *Journal of Finance*, 7(1): 77-91.

Mary, J. Francis and Rathika, G. (2015). The Single-index Model and the Construction of Optimal Portfolio with CNX PHARMA Scrip, *International Journal of Management (IJM)*, 6(1): 87-96.

Mrcela, L., Mercep, A., Begusic, S. and Kostanjcar, Z. (2017). Portfolio optimization using preference relation based on statistical arbitrage. *International Conference on Smart Systems and Technologies (SST),* Osijek. pp. 161-165.

Mroua, M. and Abid, F. (2014). Portfolio revision and optimal diversification strategy choices. *International Journal of Managerial Finance*, 10(4):537-564.

Poornima, S. and Remesh, A. (2015). Construction of optimal portfolio using Sharpe's single-index model: A study with reference to banking & IT sectors. *International Journal of Applied Research*; 1(13): 21-24.

Ramanathan, K. V. and Jahnavi, K. N. (2014). Construction of optimal equity portfolio using the Sharpe index model with reference to banking and information technology sectors in India from 2009 to 2013. *International Journal of Business and Administration Research Review*, 2(3): 122-131.

Roll, R. (1992). A mean/variance analysis of tracking error. *Journal of Portfolio Management,* 18(4): 13-22.

Sarker, M. R. (2013). Optimal Portfolio Construction: Evidence from Dhaka Stock Exchange in Bangladesh. *World Journal of Social Sciences*, 3(6): 75-87.

Shah, C. (2015). Construction of Optimal Portfolio Using Sharpe Index Model & CAPM for BSE Top 15 Securities, *IJRAR- International Journal of Research and Analytical Reviews*, 2(2): 168-178.

Sharpe, W. F. (1963). A Simplified Model for Portfolio Analysis. *The Journal of the Institute of Management Science,* 9(2): 277-293.

Singh, S. and Gautam, J. (2014). The Single-index Model & the Construction of Optimal Portfolio: A Case of Banks Listed in NSE India. *Risk Governance & Control: Financial Markets & Institutions*, 4(2): 110-115.

Sinha, P., Chandwani, A. and Sinha, T. (2015). Algorithm of construction of optimum portfolio of stocks using genetic algorithm. *International Journal of System Assurance Engineering and Management*, 6(4): 447–465.

Solimanpur, M. Mansourfar, G. and Ghayour, F. (2015). Optimum portfolio selection using a hybrid genetic algorithm and analytic hierarchy process. *Studies in Economics and Finance*, 32(3): 379-394.

Strong, R. (2008), *Portfolio Construction, Management and Protection*. 5th Edition. South Western College.

Zaimović, A., Arnaut Berilo, A. and Mustafić, A. (2017). Portfolio Diversification in the South-East European Equity Markets. *The South-East European Journal of Economics and Business,* 12(1): 126-135.